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Theoretical analysis of plastic zone of a circle crack under gigacycle fatigue regime

T. Wu [a], T. Mo [a], W. Chen [a], E. Bayraktar [b,*]
[a] Nanjing University of Aeronautics and Astronautics, College of Energy and Power Engineering, Nanjing, P.R. 210016 China
[b] School of Mechanical and Manufacturing Engineering, Supmeca/LISMMA-Paris, (EA 2336) , France
* Corresponding author: E-mail address: bayraktar@supmeca.fr

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ABSTRACT

Purpose: Different kinds of alloys used in industry for structures and engine components are subjected to very high cycle fatigue (gigacycle regime) damage under the service conditions. In this study, fatigue damage evolution of some metallic-industrial alloys was investigated on very high cycle fatigue regime and calculation of the stress intensity factor (SIF) using finite element method (FEM) was realized under the ultrasonic vibration conditions. A formula of SIF vs. Cracks size and position of the crack has been developed. In fact, calculation of the SIF under ultrasonic vibrating fatigue has to be a function of amplitude instead of nominal stress as frequently used in traditional fatigue from Woehler.

Design/methodology/approach: The specimens are tested at ultrasonic fatigue frequency of 20 kHz with a stress ratio of R=-1 (tension-compression) under load control. In order to control the displacement amplitude at the end of the amplifier was calibrated by optical sensor before each fatigue test. Failure mechanisms have been studied by means of the scanning electron microscope (SEM). The fracture origin and/or inclusions were identified by use of energy disperse analysis.

Findings: An analytical approach was validated to calculate the stress intensity factor, KI, for the specimen specially designed for Gigacycle fatigue test.

Practical implications: This heat source will be useful to position and size the small crack inside the specimen according to surface temperature variation with further heat transfer analysis. The relation between energy power and stress intensity factor helps to determine crack size and position from the temperature field on the specimen surface.

Originality/value: Based on the SIF calculation and from classical fracture mechanics, dissipated energy of the plastic zone of the crack is derived and considered as heat source when the crack initiates.

Keywords: Stress intensity factor (SIF); Finite element method (FEM); Gigacycle regime; Gigacycle fatigue test

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METHODS OF ANALYSIS AND MODELLING

1. Introduction

In manufacturing engineering, many components are subjected to high-frequency, low-amplitude, very high cycle loading in their working service. In very high-frequency (20-30-40 kHz), gigacycle fatigue of the pieces under service conditions can result in unpredictable failures of the working system. In order to explore fatigue properties and major reasons of the components collapsed in gigacycle regime, ultrasonic fatigue test had been developed and nowadays, this revolutionary technique offers an alternative and unusual testing method for generating the data necessary at very long fatigue lives behind 10^9 cycles [1-6]. Many industrial materials display a decrease continuously on fatigue strength between fatigue lives of 10^6 and 10^{10} cycles with a different slope depending on the material in gigacycle regime. In the high and low cycle ranges, the initial sites of fatigue crack are different. In case of VHCF (>10^7 cycles), the initiation sites are found at non-metallic inclusions located in the interior of the specimen; on the other hand, the initiation sites may be found at surface or just near the surface. In literature [1, 2, 4-9, 11, 13], there are many interpretations about the failure mechanism in very high cycle fatigue. The general idea is that two initiation mechanisms are acting: crack initiation is found at the surface up to 10^6 cycles, beyond 10^7 cycles, initiation site is internal of the specimen. Former studies have shown very well that the failure mechanisms in gigacycle regime can results in the effects of defects such as inclusions, pores and internal microstructural or natural metallurgical defects on the crack initiation [4-6, 9-13]. Actually, the initiation site, which depends on a defect, is not always in the interior of the specimen at this level of cycles, as claimed by classical damage mechanism, since the probability of finding a defect at the surface depends on the number of defects for a given volume of the specimen. Additionally, the cause of the initiation site is not only the inclusion but also the pores, the nature of microstructure and/or the metallurgical defects.

Nowadays, the studies carried out by an infrared camera show in-body crack initiation, to observe the temperature variation on the specimen surfaces [8-9]. Under symmetrical vibration, stress and strain form a loop and a plastic zone occurs under the tension-compression loading. This is a dissipated energy which is the heat source in the body of specimen and can be seen on the specimen surface. The experimental results were shown in Figure 1 [5]. First of all, this figure shows very well that the temperature increases only at the level of fracture of the specimen.

The crack is normally initiates from an inclusion. When it grows, it will release heat.

This paper aims a detail theoretical research based on the experimental results. An analytical approach is given to calculate the position and the size of the crack that can be easily compared with that of the temperature field recorded by the infrared camera. As well determined by former studies [6, 4, 2] that there is a small plastic zone at the tip of crack called fish eye. From this simple idea, the stress intensity factor (SIF) can be calculated with a new approach that is presented in the present paper. So, the size of the zone can be calculated if the Stress Intensity Factor (SIF) is known. In this case, there is no formula of SIF calculation under vibration regime. Giving some experimental results, this paper gives firstly a comprehensive analytical approach to determine SIF, and then discusses the plastic strain at the tip micro crack and calculates the strain energy under gigacycle fatigue regime.

2. Experimental conditions

The specimens are tested at ultrasonic fatigue frequency of 20 kHz with a stress ratio of R=-1 (tension-compression) under load control. In order to control the displacement amplitude at the end of the amplifier was calibrated by optical sensor before each fatigue test.

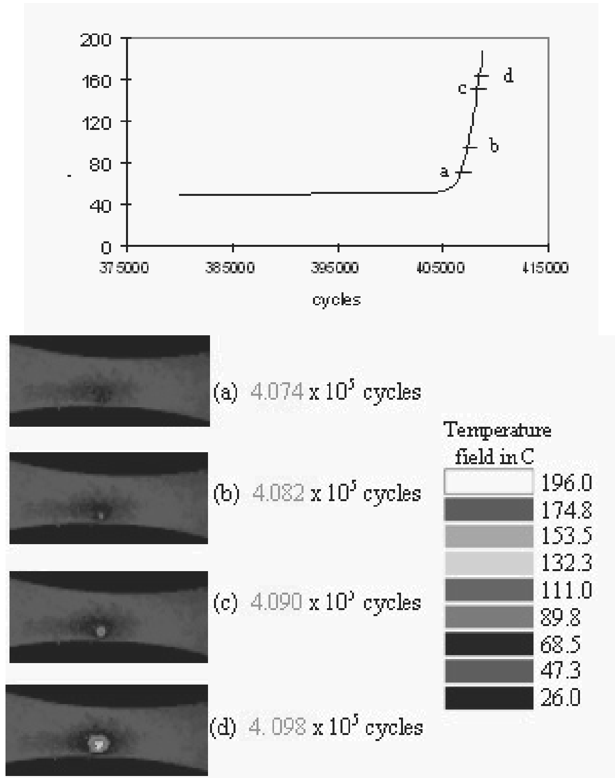


Fig. 1. Temperature field just before rupture; 42CD4 steel with stress amplitude of 460 MPa [8, 5]

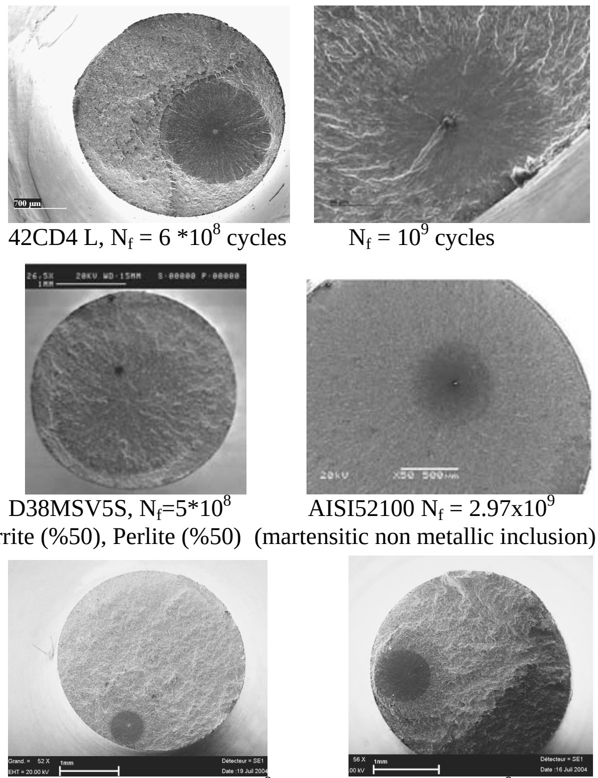
All of the samples were tested as received conditions without polishing. The temperature of the central part of the specimen was kept constant at room temperature by a special temperature cooling system with compressed air (~10 °C) during the test. It can arrange the level of the cooling rate automatically. The mechanisms of fatigue failure have been studied by means of analysis of fractography on the scanning electron microscope (SEM). The fracture origin and/or inclusions were identified by use of energy disperse analysis. All the details for ultrasonic fatigue test are found in the papers of [6, 4, 9].

3. A new approach to calculate the stress intensity factor

Our earlier works showed that the inside crack initiation by forming fish eye is a typical damage phenomenon in cyclic loading in gigacycle regime [1, 4, 6, 9-10]. Figure 2 shows typical fracture surfaces and the formation of fish eyes in different materials. All fish eyes are almost in form of perfect circle.

Thus, Finite Element Method (FEM) was used to calculate the stress intensity factor (SIF) from a circle crack. In the FEM analysis, the singular elements were used and a solution was given under vibration model that is used in ultrasonic fatigue. After determining SIF, the value of SIF, the volume of the plastic zone around a circle crack is found by calculation by formula as usually made in fracture mechanics. Internal crack initiations have been located from defects, such as, non-metallic inclusions, micro-cracks and microstructural defects (residual austenite, bainite and ferrite). Fish-eye shapes were often present in internal failures.

An hour glass shape of specimen has been used in all of the tests (Figure 3a) and it had a proper frequency of 20 kHz and tension-compression vibration model. The maximum nominal stress was in the centre section of the specimen, and stress field is designed symmetrical about the centre section. A quarter of specimen cut from half circular crack has been meshed for FEM analysis with an inside crack (Figure 3b) at the small section. Singular elements have been used at the tip of crack. Calculation of stress intensity factor, SIF was very sensitive to the boundary conditions and the size of singular elements [2]. All of the nodes in cut surfaces except those in crack surface have been constrained only by a null perpendicular displacement. The size of singular elements has been used as small as possible.



42CD4 L, N_f = 6 *10^8 cycles $N_f = 10^9$ cycles

D38MSV5S, N_f=5*10^8 AISI52100 $N_f = 2.97 \times 10^9$

Ferrite (%50), Perlite (%50) (martensitic non metallic inclusion)

SAE 8620 $N_f = 9.5457 \times 10^9$ $N_f = 2.31 \times 10^9$

(75%ferrite-25%perlite)

Fig. 2. Fracture surfaces of gigacycle fatigue specimens with fish eyes [6]

According to the fracture mechanics, SIF varies along the circle of crack depending on the size and the position of the crack. However, according to the results of calculation, SIF does not have significant change along the circle of crack.

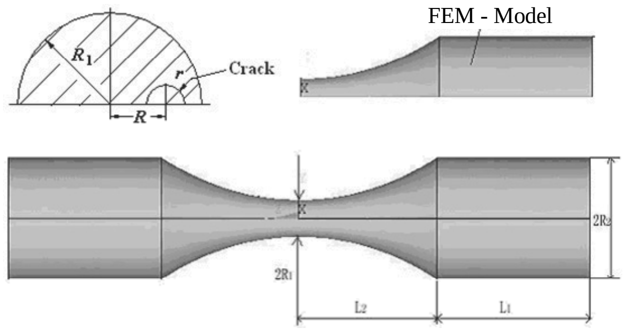


Fig. 3a. Specimen design and FEM considering of nomenclature to explain the size and geometrical position of the crack

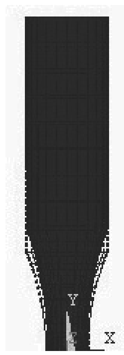


Fig. 3b. Mesh figure for the finite element analysis (FEM)

In this case, it can be considered approximately SIF as only a function of size and position. Mechanically, the specimen vibrates in model of axial tension-compression and the following equation can be used in FEM calculation [1, 3]:

$$[K]\{u\} = \omega^2 [M]\{u\} \tag{1}$$

where, [K], [M] are rigidity and mass matrix respectively, {u} is displacement vector, and $\omega=2\pi f$, f is frequency.

SIF can be obtained from {u} [10] and it has to be proportional to vibration amplitude A_0 measured by a displacement (optical) sensor at the end of the specimen. It can be described by the following function.

$$K_1 = \frac{E}{1-\nu^2}\sqrt{\frac{\pi}{r}}A_0 f\left(\frac{r}{R_1}, \frac{R}{R_1}\right) \tag{2}$$

Figure 3 illustrates the geometrical position of the crack considered in FEM calculation. Here, r is the radius of the crack, R_1 is the radius of the minimum section of the specimen, and R is the eccentric distance of the crack and also $f\left(\frac{r}{R_1}, \frac{R}{R_1}\right)$ presents the geometric factor as a function of two dimensionless parameters: size and position.

First conclusion in FEM analysis has given that small difference is found in K_I value around the crack circle. This explains why those fish eyes observed in SEM (Fig. 2) are almost circle with the defect in centre; crack growth rate is almost the same along all directions. Independence of direction, this conclusion allows simplify K_I expression to use an average K_I for a circle crack.

For a given specimen, material, crack size and fixed crack position, SIF can be resolved through FEM. Then, numerical values of $f\left(\frac{r}{R_1},\frac{R}{R_1}\right)$ are calculated based on the equation (2). The least square method is used to determine the expression (3) of $f\left(\frac{r}{R_1},\frac{R}{R_1}\right)$

$$f\left(\frac{r}{R_1},\frac{R}{R_1}\right)=-0.169\left(\frac{r}{R_1}\right)+8.25\left(\frac{r}{R_1}\right)^2-66.14\left(\frac{r}{R_1}\right)^3+145.9\left(\frac{r}{R_1}\right)^4$$
$$+1.22\left(\frac{rR}{R_1^2}\right)-32.23\left(\frac{r^2R}{R_1^3}\right)+243.8\left(\frac{r^3R}{R_1^4}\right)-526.9\left(\frac{r^4R}{R_1^5}\right)$$
$$-1.02\left(\frac{rR^2}{R_1^3}\right)+26.63\left(\frac{r^2R^2}{R_1^4}\right)-192.6\left(\frac{r^3R^2}{R_1^5}\right)+404.2\left(\frac{r^4R^2}{R_1^6}\right) \quad (3)$$

Figure 4 shows that $f\left(\frac{r}{R_1},\frac{R}{R_1}\right)$ is the same for different metals whose sound propagation speed $\sqrt{\frac{E}{\rho}}$ varies slightly, between 4.9 and 5.25 for a $\frac{R}{R_1}$ constant where E is Young's module and ρ is density. Thus, it does not give a difference from one material to other. In addition, this can be the same conclusion as that in our previous studies [1, 10]. Equation (2) can be used for any metallic specimen having the same form.

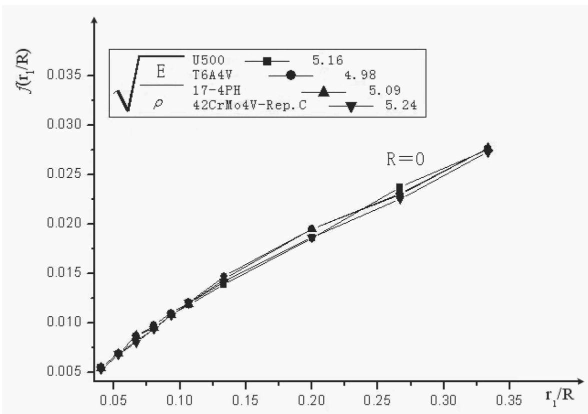


Fig. 4. Correction factor of K defined as a function of crack size for different metals

However, in this study, it will be useful to compare of K_I value at dynamic and conventional conditions (Figure 5). In conventional equation, K_I is quantified with static loading force or nominal stress calculated by static loading divided by the initial surface because the loading force is easy to be measured and controlled by the test machine and this kind of tests is in general under constant force (or amplitude of force). However, in ultrasonic-vibrating fatigue tests, displacement is directly measured by an optical or other kind of displacement sensor and the nominal stress in specimen centre can be calculated from the measured displacement. Vibrating test machine controls the vibration amplitude as constant. That means that vibrating fatigue tests are equivalent to displacement constant tests in conventional cases where the nominal stress decreases with crack length. In case of "constant nominal stress", the value will be as $K_I''(a/w) > 0$ (second order derivative) and in case of "constant displacement", the value will be as $K_I''(a/w) < 0$. Thus, it is evident that two curves are totally different and it is not correct to use nominal stress based on the displacement amplitude to quantify K_I. On the other hand, in the two kinds of tests, measured parameters are force for conventional and displacement for vibrating. Use of directly measured parameters to calculate SIF is natural way and gives less error.

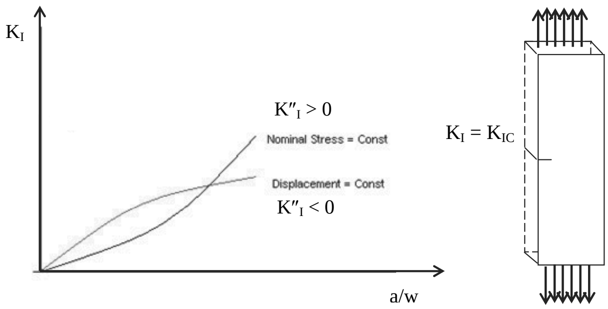


Fig. 5. Comparison of K_I value in dynamic and conventional conditions; K_I vs crack size

4. Study of the fatigue dissipated energy

4.1. Plastic zone at the tip of the crack

Comparative study related to the form and the dimension of the plastic zone at the tip of the crack requires explaining the deformation mechanism around it. The form of this plastic zone depends not only on the state of stresses but also the form of the involved crack. In fact, the analysis of the efficient stresses at the crack tip requires the application of Von-Misses criteria to determine the size of the plastic zone and also to calculate its volume.

Thus,

$$\sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]}=\sigma_s \quad (4)$$

where, σ_s is the yield stress of the material obtained during uniaxial tension.

It means that yielding will occur when the differences of stresses on the left side of the equation (4) exceed the yield stress in uniaxial tension, σ_s.

As known, in very high cycle fatigue (VHCF) regime, the crack inside of the specimen propagates from inside to outside. Additionally, there is always a plane-strain condition for a specimen used in VHCF regime. Thus, the plane-strain values of critical stress results in the plastic deformation at the crack tip. In another words, a plastic zone will exist at the crack tip. The three principal stresses are given as follows;

$$\left.\begin{matrix}\sigma_1\\ \sigma_2\end{matrix}\right\} = \frac{K_I}{\sqrt{2\pi r_1}}\cos\frac{\theta_1}{2}\left(1 \pm \sin\frac{\theta_1}{2}\right) \tag{5}$$

$$\sigma_3 = \sigma_{zz} = \upsilon\left(\sigma_1 + \sigma_2\right) = 2\upsilon\frac{K_I}{\sqrt{2\pi r_1}}\cos\frac{\theta_1}{2} \tag{6}$$

where, σ_{zz} is one of the principal stresses. Then the size (circle) of the plastic zone is explained as the following;

$$r_1 = \left(\frac{K_I^2}{2\pi\sigma_s^2}\right)\cos^2\frac{\theta_1}{2}\left[\left(1-2\upsilon\right)^2 + 3\sin^2\frac{\theta_1}{2}\right] \tag{7}$$

It means that the plastic zone takes a cylindrical shape. By combining the equations (4) to (7) and assuming that the Poisson's ratio is always equal to 0.3, integration on the size (circle) of the plastic zone allows calculating its volume:

$$V = 2\int_0^{\pi}\int_0^{\cos^2\frac{\theta_1}{2}\left(0.16+3\sin^2\frac{\theta_1}{2}\right)\left(\frac{K_I^2}{2\pi\sigma_s^2}\right)} 2\pi\left(r + r_1\cos\theta_1\right)r_1 dr_1 d\theta_1$$
$$= 6.895\times10^{-4}K_I^6\sigma_s^{-6} + 1.4\times10^{-1}rK_I^4\sigma_s^{-4} \tag{8}$$

where r_1 is the distance between the centre of the crack and the centre of the specimen, K_I is the stress intensity factor.

This equation shows that the plastic volume extends with the increasing of K_I and decreasing with σ_s. In fact, the smaller value of r_1 for plane strain is consistent with the fact that the triaxiality in stress field in plane strain suppresses the extent of the plastic deformation.

4.2. Specific strain energy of the plastic zone under cyclic solicitation, fatigue loop

Calculation is based on the fatigue stress strain (hysteresis) loop shown in the Figure 6 for constant strain cycling. This loop can be considered as a parallel quadrilateral. The width of the hysteresis loop will depend on the level of cyclic strain.
The surface area of this quadrilateral should be equal to:

$$u = 2\sigma_s\Delta\varepsilon = 2\sigma_s(\varepsilon_{ef} - \sigma_s/E)$$

Therefore, the following derivation intends to determine ε_{ef}. It means that the specific strain energy or the absorbed energy by a unit volume of the material as a result of crack tip plasticity is represented by the area surrounded by σ vs. ε loop. This loop can be shown schematically by tracing the σ vs. ε points generated during cyclic solicitation, where the strain passes from minimum to maximum and then arrive to the minimum (Without strain hardening of the material, this loop is repeated whereas cyclic load continues, etc.).

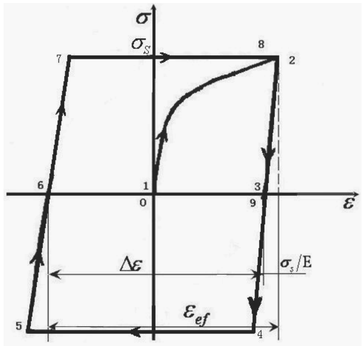


Fig. 6. Stress–strain loops under monocycle for constant strain cycling

Under very high frequency in ultrasonic fatigue (20-30 kHz), the storage of the strain energy is very rapid that induce the failure very rapidly just after the appearance of a crack in the specimen [1-3 7].

According to the Fig. 6, Illustration in 3D of the plastic zone, the application of generalized Hooke's law allows setting the relations between the principal strains with principal stresses:

$$\left.\begin{aligned}
\varepsilon_1 &= \frac{1}{E}\left[\sigma_1 - \upsilon\left(\sigma_2+\sigma_3\right)\right] = \frac{K_I}{E\sqrt{2\pi r_1}}\cos\frac{\theta_1}{2}\left(0.52 + 1.3\sin\frac{\theta_1}{2}\right)\\
\varepsilon_2 &= \frac{1}{E}\left[\sigma_2 - \upsilon\left(\sigma_1+\sigma_3\right)\right] = \frac{K_I}{E\sqrt{2\pi r_1}}\cos\frac{\theta_1}{2}\left(0.52 - 1.3\sin\frac{\theta_1}{2}\right)\\
\varepsilon_3 &= \frac{1}{E}\left[\sigma_3 - \upsilon\left(\sigma_2+\sigma_1\right)\right] = 0
\end{aligned}\right\} \tag{9}$$

where E is Young's modulus and υ = 0.3 is the Poisson's coefficient.

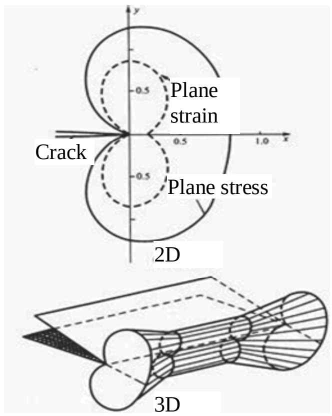


Fig. 7. 2D and 3D illustration of the crack and the plastic zone

In practical point of view, elastic strain can be assigned to elastic-plastic strain to obtain effective strain as the following:

$$\varepsilon_{ef} = \frac{\sqrt{2}}{3}\left[\left(\varepsilon_1-\varepsilon_2\right)^2 + \left(\varepsilon_2-\varepsilon_3\right)^2 + \left(\varepsilon_3-\varepsilon_1\right)^2\right]^{1/2}$$
$$= \frac{2K_I}{3E\sqrt{2\pi r_1}}\cos\frac{\theta_1}{2}\sqrt{0.2704 + 5.07\sin^2\frac{\theta_1}{2}} \tag{10}$$

Thus, hysteresis strain $\Delta\varepsilon$ in the plastic zone is written as the following:

$$\Delta\varepsilon = \varepsilon_{ef} - \sigma_s / \mathrm{E}$$

It allows calculating the equivalent strain energy

$$u = 2\sigma_s \Delta\varepsilon = 2\sigma_s(\varepsilon_{ef} - \sigma_s/\mathrm{E})$$

By integration of this equation for the total plastic zone, dissipated energy can be calculated in the plastic zone for one cycle;

$$U = \iiint_\Omega u dv$$
$$= 2\int_0^\pi \int_0^{\cos^2\frac{\theta_1}{2}\left(0.16+3\sin^2\frac{\theta_1}{2}\right)\left(\frac{\mathrm{K}_\mathrm{I}^2}{2\pi\sigma_s^2}\right)} 2\sigma_s(\varepsilon_{ef} - \frac{\sigma_s}{\mathrm{E}})2\pi\left(r + r_1\cos\theta_1\right)r_1 dr_1 d\theta$$
$$= 0.81\sigma_s^{-4}\mathrm{E}^{-1}\cdot\left(6.81\times10^{-5}\mathrm{K}_\mathrm{I}^6 + 5.38\times10^{-2} r\sigma_s^2\mathrm{K}_\mathrm{I}^4\right) \quad (11)$$

where r is the crack radius (m), σ_s is the yield stress of the material (MPa), E is the elastic modulus (MPa) and K_I is the stress intensity factor, SIF (MPa√m).

For very high frequency in fatigue, total strain energy in the plastic zone depends also on the frequency and on the elapsed time. If the crack radius, r is 8.10^{-5}m, by applying equation (11) for different industrial materials used in this study (Table 1), results have been presented in Figure 8 drawing the evolution of plastic strain energy, U as a function of stress intensity factor, K.

Table 1.
Mechanical properties of materials used in this study

Materials	E (GPa)	ρ (Kg/m^3)	$\sigma_{0.2}$ (MPa)	r (mm)
42CD4 (this study)	205	7870	1460	0.08
AISI 52100 (this study)	210	7860	1158	0.08
D38MSV5S (this study)	208	7850	608	0.08
Perlitic steel XC70 (this study)	210	7850	513	0.08
Cast iron GS51 (this study)	169	7100	460	0.08
Aluminium 2-AS5U3G-Y35 (this study)	72	2700	182	0.08
Ti6Al4V	112	4700	825	0.08
GH118	220	8970	838	0.08
LY12	72	2700	380	0.08

By application of formula (11) for a given crack radius of r=0.08mm and for K_I = 25MPa√m, it can be found U=1.4×10^{-4}J for LY12. In the high frequency of vibration of 20kHz, stored plastic strain energy in one second in the plastic zone is found: Q=$20000\times1.4\times10^{-4}$=2.8 Watts. This value is relatively consistent and they give a clear idea about releasing heat energy level. For example, a cast iron grade of GS51 can release more heat energy than other alloys such as XC70, D38, 42CD4, under the same K_I (Fig.8 using material properties in Table 1). That means a tender material release more heat under the same K_I.

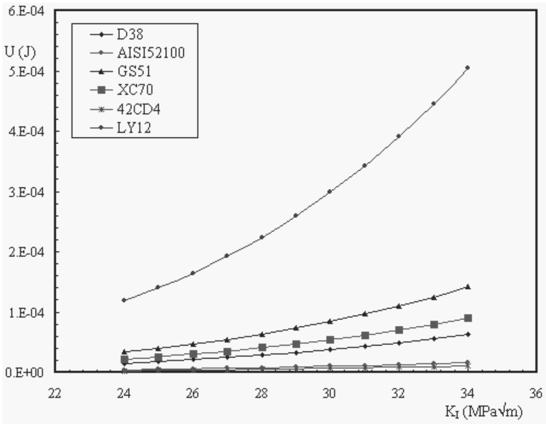


Fig. 8. Evolution of plastic strain energy, U as a function of stress intensity factor, K_I

However, the dissipated energy discussed here belongs to the cyclic plastic strain at micro crack tip and this level is found in order of 10 Watts in our test conditions that is a reasonable value for the ultrasonic testing machine.

5. Conclusions

In this paper, an analytical approach was validated to calculate the stress intensity factor, K_I, for the specimen specially designed for Gigacycle fatigue test. Equations (2) and (3) are suitable for metal specimens calculated variation of K_I around the circle is not significant and can be neglected. Observations of the fracture surfaces by means of the SEM provide evidence this conclusion. Internal cracks of the specimen are geometrically circle and the inclusion that initiates the crack was observed in the center of the circle. Calculation of K_I under vibrating fatigue must be quantified with amplitude measured in tests.

According to the fracture mechanics, size of plastic zone at the tip of the crack depends on K_I. Equation (11) gives the dissipated energy in the plastic zone for one cycle under fatigue. The dissipated energy power of a crack is in order of 10 Watts which is comparable with the ultrasonic fatigue test machine capacity used in this study. This energy power is the heat resource observed by infrared camera. The relation between energy power and stress intensity factor helps to determine crack size and position from the temperature field on the specimen surface.
In a further study, an inverse method will be used to calculate and the location of the heat resource.

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